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# Wash Waves Generated by Ships Moving on Fairways of Varying Topography

Tao Jiang[1], Rupert Henn[2] & Som Deo Sharma[2]

([1]VBD - European Development Centre for Inland and Coastal Navigation, [2]Institute of Ship Technology and Transport Systems, Mercator University, Duisburg, Germany)

## ABSTRACT

Using the computer program BEShiWa, based on extended Boussinesq's equations for the far-field flow and slender-body theory for the near-ship flow, the wash waves generated by ships were investigated taking account of varying topographies. It appears that for the time being the shallow-water approximation based on Boussinesq-type equations is a useful method for combining all effects associated with the nonlinear and unsteady nature on the one hand and the large-domain feature on the other hand of wash problems. A good agreement between computations and measurements was achieved for waves far from the ship in a rectangular channel as well as for 2D wave propagation over an uneven bottom. Furthermore, it was found that the propagation of wash waves depends significantly on bottom topography, ship speed, and motion history.

## INTRODUCTION

Recently, increased attention is being paid to wash waves by ferry operators, naval architects, marine and environmental consultants, and port and waterway authorities. Such wash waves can affect the safe operation of other floating bodies near the shoreline or the bank and endanger human life on beaches. Moreover, they can cause environmental damage, for instance, bank erosion.

Strong wash waves can be generated by a fast ship at high speeds or by a large ship at moderate speeds, operating on a near-shore fairway or on an inland waterway. The resulting wash-wave system is basically nonlinear due to the wave characteristics in shallow water and usually unsteady due to the non-uniform seabed or inland waterway topography. Moreover, the major part of the wash wave system consists of divergent waves which can travel over a long distance without loss of energy until close to the shore.

Due to the great interest in wake-wash effects, a considerable amount of research effort has been devoted to the wash problem during the last few years. In model experimental studies the focus has been on designing low-wash ships and acquiring reliable data for validation, see, e.g., Zibell and Grollius (1999), Macfarlane and Renilson (1999), and Koushan et al.(2001). Also full-scale measurements have been taken, aiming at deriving recommendations for safe ship operation as well as finding possibilities of ship monitoring in sensitive operational areas, see, e.g., Feldtmann and Garner (1999), and Bolt (2001). In numerical simulations the focus has been on developing efficient methods. For a ship moving on water of uniform depth the linear theory can be applied usefully in the subcritical and the supercritical speed range, see, e.g., Doctors et al.(2001). For these steady cases a steady nonlinear free-surface panel method can also be used, see, e.g., Raven (2000). The wave generation by a ship moving in a channel at a transcritical speed, on the other hand, can be well predicted using Kadomtsev-Petviashvili (KP) type equations, see, e.g., Chen and Sharma (1995), where the near-ship flow is approximated by an extended slender-body theory. This KP approximation has been extended by Chen and Uliczka (1999) for ships moving in natural waterways with transversally varying water depth. But a basic restriction of the KP equation is that it is not valid for truly unsteady cases, caused, for instance, by varying topography along the ship's track. A more general shallow-water approximation are equations of Boussinesq type, which are valid for almost arbitrarily unsteady cases. In Jiang (1998) a set of modified Boussinesq's equations, which are valid not only for long waves but also for waves of moderate length, was applied to compute ship waves in shallow water, using slender-body theory to approximate the near-ship flow. In Yang et al.(2001) Boussinesq's equations based on a suitable reference level were used for computing ship waves in shallow water but the near-ship flow was represented by a mean transverse velocity from slender-body theory instead of using the near-field velocity at the reference level. A hybrid approach, comprising the coupling of a steady nonlinear panel method for the near-ship flow to a Boussinesq solver for the far-field wave propagation, has been introduced by Raven (2000). However, it is useful only for steady problems.

It should be noted here that due to the non-linear and unsteady nature as well as the large-domain feature of the wash problems, they can be neither solved well by the linear wave theory nor approximated efficiently by a nonlinear singularity-method, even less by a finite-volume method due to the huge computational domain required. To cover all effects mentioned above an efficient method for the time being seems to be a shallow-water approximation based on Boussinesq-type equations, in which the 3D governing equations for the inviscid fluid are first treated analytically in the vertical direction and the resulting 2D equations then solved numerically in the horizontal plane. In an extensive study by Jiang (2001) a computer program BEShiWa, standing for Boussinesq's Equations for Ship Waves, has been developed with the following features:

- extension of the shallow-water equations of Boussinesq type to longer and shorter waves over an uneven bottom,
- inclusion of the near-ship flow into the shallow-water equations either through the law of conservation of mass or through a free-surface pressure distribution equal to the hydrostatic pressure on the hull bottom or through a unified shallow-water theory,
- implementation of suitable boundary conditions, and
- application of numerically efficient and robust methods.

In the present study, we focus on the wash-wave systems generated by a Panmax containership and a fast inland passenger-ferry. Special attention is paid to the interaction between ship waves and bottom topography, aiming to find practical criteria for safe ship operation (speed and distance to shoreline) as well as for fairway maintenance (dredging depth and frequency) with regard to the wash effects.

## BRIEF MATHEMATICAL DESCRIPTION AND NUMERICAL APPROXIMATION

### Coordinate System

For describing the flow generated by a ship sailing in shallow water over a general topography, a right-handed earthbound coordinate system *Oxyz* is used. The origin *O* lies on the undisturbed water-plane. The *x*-axis points in the direction of ship's forward motion; the *z*-axis, vertically upwards.

### Field Equations

Considering water as incompressible and inviscid, the wave generation by ships in shallow water can be approximated by the well-established shallow-water wave theory, see Jiang (2001) for a review. Assuming that the water depth is small in comparison to the wave length and that the wave amplitude is small in comparison to the water depth, the wave field can be well described by shallow-water equations of Boussinesq type. In the present study, Boussinesq's equations based on the mean horizontal depth-averaged velocity for an uneven bottom, without additional terms for correcting the dispersion relation, are applied:

$$\zeta_t + (\zeta_x + h_x)u + (\zeta + h)(u_x + v_y) + (\zeta_y + h_y)v = 0$$

$$u_t + uu_x + vu_y + g\zeta_x - \frac{h}{2}(h_{xx}u_t + 2h_x u_{tx} + hu_{txx} + h_{xy}v_t + h_y v_{tx} + h_x v_{ty} + hv_{txy}) + \frac{h^2}{6}(u_{xxt} + v_{xyt}) = 0$$

$$v_t + uv_x + vv_y + g\zeta_y - \frac{h}{2}(h_{xy}u_t + h_x u_{ty} + h_y u_{tx} + hu_{txy} + h_{yy}v_t + 2h_y v_{ty} + hv_{tyy}) + \frac{h^2}{6}(u_{txy} + v_{tyy}) = 0$$

These were first derived by Peregrine (1967).

Herein, $h(x,y)$ is the water depth, $\zeta(x,y,t)$ the wave elevation, $u(x,y,t)$ and $v(x,y,t)$ the depth-averaged velocity components in the $x$ and $y$ directions, respectively, $t$ the time, and $g$ the acceleration due to gravity. For this set of nonlinear partial differential equations there exists no analytical solution. Therefore, it has to be solved numerically.

### Boundary Conditions

On the truncation boundaries of the computational domain sufficiently far from the ship the Sommerfeld radiation condition

$q_t + \sigma q_x = 0$ or $q_t + \sigma q_y = 0$

is applied, where $q$ stands for each of the variables $\zeta$, $u$ and $v$, and $\sigma$ ensures the local outgoing characteristic of the governing equations on the boundary in question, for instance, $\sigma = \sqrt{gh}$ ahead of the ship and $\sigma = -\sqrt{gh}$ behind the ship.

On vertical channel sidewalls, if any, the condition of no-flux or, equivalently, perfect reflection holds.

**Initial Conditions**

In compliance with the unsteady nature of the flow, the ship is assumed to start from rest and accelerate uniformly to a final velocity like in a model towing tank. As may be expected, the final wave system is found to be influenced by the acceleration rate, especially in case of trackwise varying topography. This is because the waves caused by the accelerating ship with a rather arbitrarily assumed starting point can be reflected by the bottom topography and then interact with the waves generated by the ship at steady speed.

**Approximation of the Near-Ship Flow**

The main interest in the present study lies in the wave propagation far from the ship. So a slender-body theory is applied to approximate the near-ship flow. Starting from the general formulation of the depth-averaged mean transversal velocity for a slender body in Jiang (2001), and additionally taking account of the asymmetric effect caused by nonuniform channel topography, the boundary condition on the longitudinal ship-centerline (the mathematical dividing line between the near-field and the far-field) relevant to the far-field flow reads

$$v\big|_{y\to 0^{\pm}} = \mp \frac{1}{2(h+\zeta_0)}[(V-u_0)\zeta_0 B_x + hBu_{0x} + VS_x - (u_0 S)_x] + v_0$$

with the port-starboard mean values of the longitudinal velocity component $u_0 = \dfrac{u\big|_{y\to 0^+} + u\big|_{y\to 0^-}}{2}$, of the transversal velocity component $v_0 = \dfrac{1}{2C(x)}\int_{x_{\text{stern}}}^{x_{\text{bow}}} (u\big|_{y\to 0^+} - u\big|_{y\to 0^-})dx$, and of the wave elevation $\zeta_0 = \dfrac{\zeta\big|_{y\to 0^+} + \zeta\big|_{y\to 0^-}}{2}$. The hull sectional area is denoted by $S(x)$, and the beam by $B(x)$. The sectional blockage coefficient $C(x)$ can be calculated by a 2D boundary element method in advance, see, e.g., Taylor (1973). $x_{\text{bow}}$ and $x_{\text{stern}}$ are the longitudinal positions of the bow and stern, respectively. Moreover, the Kutta condition is implemented at the stern through

$$u\big|_{y\to 0^+} = u\big|_{y\to 0^-}$$

Physically, it means that the longitudinal velocities on the two sides of the hull have to be identical at the ship stern to ensure that there is no pressure jump.

**Numerical Solution Technique**

To solve this initial-boundary value problem governed by Boussinesq's equations, an implicit Crank-Nicolson scheme is implemented as usual. But it encounters some difficulties arising from the nonlinear terms and the linear high-order terms. The developed solution technique comprises:

- Crank-Nicolson scheme of high-order accuracy for the time and space discretization,
- approximation of the state values of the nonlinear terms by means of Taylor series expansion,
- SOR iterative solution of the resulting sparse equation system,
- overrelaxation to accelerate the convergence, and
- local and global filtering to suppress numerical oscillations and instabilities.

## RESULTS AND DISCUSSION

**Example Ships**

Two example ships are investigated in the present study. One is an inland passenger ferry that operates on the river Rhine and in a sheltered region of varying water-depth; this aims at finding suitable criteria for fast-ship operations. The other is a Panmax container ship; this is intended to predict the wash-waves typical of large ships operating on a near-shore fairway or near a harbor. The main dimensions of both ships are listed in Table 1.

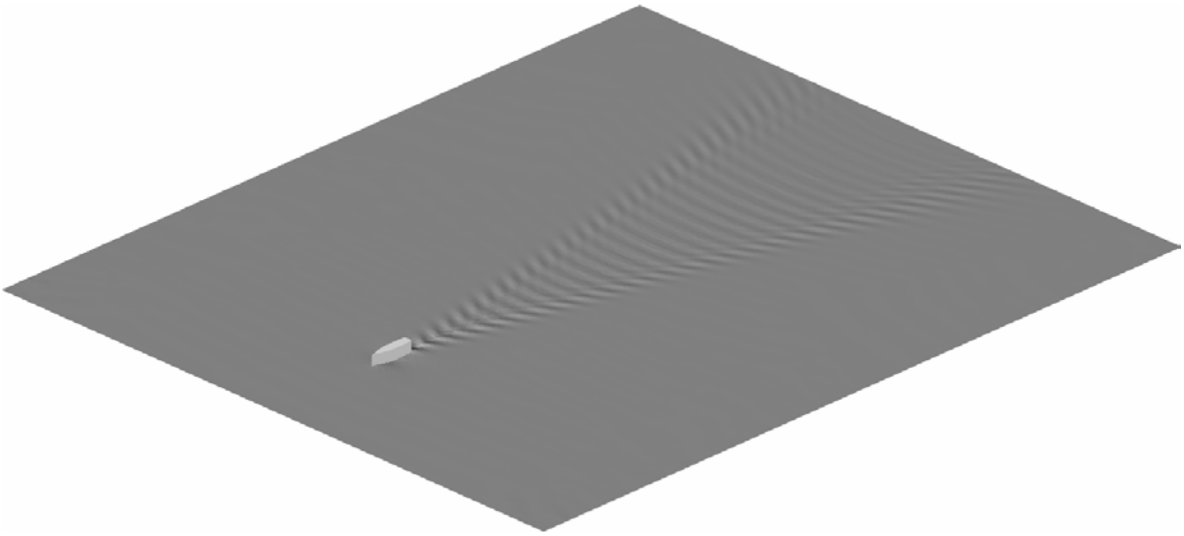
**Table 1:** Main dimensions of investigated ships

| | Inland Passenger Ferry | Panmax Container Vessel |
|---|---|---|
| $L_{WL}$ | 39.3 m | 280 m |
| $B$ | 8.8 m | 32.2 m |
| $T$ | 1.2 m | 11 m |

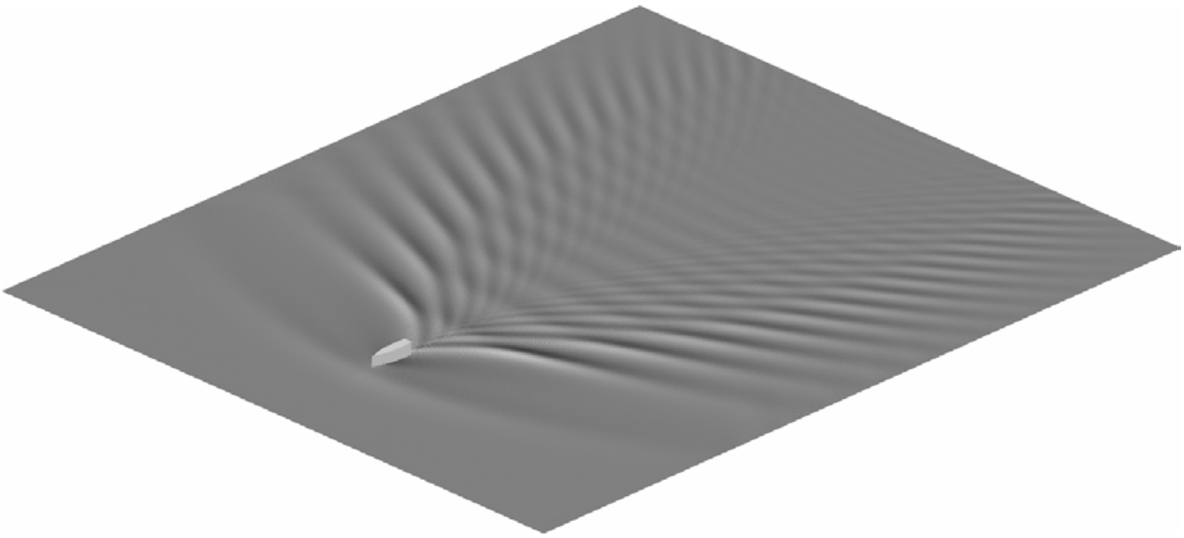
**Representative Waves in a Large Shallow-Water Region**

To demonstrate the capability of the computer program BEShiWa to predict ship waves over a huge computational domain, Figure 1 shows three representative wave systems generated by the subject inland passenger-ferry moving in an unbounded shallow-water region of uniform depth. The computational domain was of 17.5 ship lengths long and 7.5 ship lengths wide, taking advantage of transverse symmetry. The grid size was 1m × 1m, yielding a total of 210,000 grid points. The CPU time required for a typical run was about 23.5 hours for 4,500 time steps.

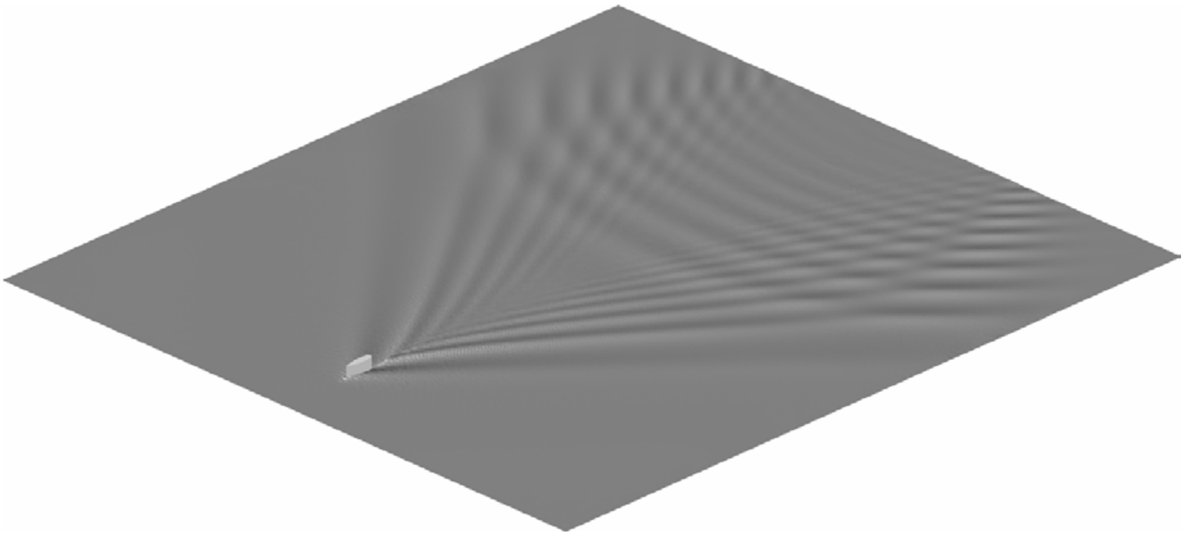
At a subcritical speed, $F_{nh} = 0.7$ in graph (a), the wave system is steady and close to a Kelvin-Havelock wave pattern with pronounced transverse waves. At critical speed, $F_{nh} = 1.0$ in graph (b), the wave system is characterized by significant divergent waves. Long-time simulations showed that no asymptotic steady state could be reached at transcritical speeds. For the same speed in a (finite-width) channel, so-called solitons, which are perfect transverse waves propagating ahead of the ship, were generated in accordance with observations in model tanks and full-scale, see, e.g., Jiang (2001). Similar unsteady response to steady excitation has been observed in other nonlinear problems. In fact, for a nonlinear system governed by Boussinesq's equations, there is no guaranty that the asymptotic solution would be steady and independent of the initial conditions. At a supercritical speed, $F_{nh} = 1.5$ in graph (c), the final wave system comprises only divergent waves, no initial transverse waves generated during the acceleration phase could keep up with the ship. The asymptotic wave system is again steady relative to the ship.



a) $F_{nh} = 0.7$



b) $F_{nh} = 1.0$



c) $F_{nh} = 1.5$

**Figure 1:** Representative wave systems generated by the subject inland passenger-ferry in shallow water

**Validation of Waves at a Large Distance from the Ship**

To validate the computational results from BEShiWa, especially at a large distance from the ship, Figure 2 compares the computed wave records (dashed lines) with those measured (solid lines) in the Duisburg Shallow-Water Towing Tank (VBD) for the inland ferry model. At the design speed of $F_{nh} = 0.873$ the following observations can be made: (i) The agreement is quite satisfactory near the ship ($y$ = 6 m) and pretty good ahead of the ship. (ii) The consistently improving agreement (in both amplitude and phase) with increasing proximity to the channel sidewall demonstrates the usefulness of the present solution method for predicting far-field wash-waves. (iii) The relatively large discrepancy in the ship's wake may have been caused by the running submergence of the transom stern which was not explicitly accounted for in the present computer program. Luckily, transverse waves on the ship's track are not relevant to the wash problem. (iv) Multiple upstream solitons ahead of the ship are not visible. However, other computations have shown that shape and speed of the wave ahead of the ship do depend on the initial acceleration pattern.

**Influence of Channel Section Shape on Wash Waves**

Figure 3 shows wave patterns generated by the inland passenger-ferry moving at critical speed (referred to the water depth along the channel centerline). Three different channel section shapes are presented to display the influence of transversally varying bottom topography on wash waves, namely, a rectangular channel section in graph (a), a trapezoidal channel section in graph (b), and a polygonal section consisting of a deepened fairway in a shallow channel in graph (c), all three of the same depth at the centerline and the same width overall. In all cases the ferry runs along the channel centerline. It is seen that unlike the rectangular channel (a) perfect solitary waves could be generated neither in the trapezoidal channel (b) nor in the deepened fairway (c), although a perfect-reflection condition was implemented on the sidewalls in all cases. An important observation is that the highest waves occur either near the sidewall in the trapezoidal channel (b) or in the shallow region beside the deepened fairway (c). These high waves could affect the safe operation of other floating bodies near the bank or possibly cause bank-erosion.

**Effects of Initial Ship Acceleration**

Since wave elevation and wave energy both propagate in shallow water at the same velocity, an initial disturbance would also travel at the same speed. This leads to an interaction of waves generated in the initial acceleration phase with those later generated by the ship at its asymptotic speed. Due to the possible scattering of initial waves by the longitudinal bottom-topography, the resulting wave pattern could depend on the ship acceleration or deceleration. To examine this effect, two simulations were performed for the subject container ship moving on a near-shore fairway. The contour plot of the investigated fairway is given in Figure 5. The wave patterns generated are shown in Figure 6 (a) and (b) for the case of slow and fast acceleration, respectively. As expected, the larger the acceleration, the higher the initial waves observed ahead of the main wave system generated by the steady ship motion. Within the main wave system, the initial acceleration influences strongly the so-called primary wave but only weakly the trailing waves. This phenomenon can be clearly observed in Figure 7 showing wave records taken by four simulated wave probes (locations marked in Figure 5). Moreover, an interaction of waves generated in the acceleration phase with those later generated by ship at constant speed can be noticed in the transition between the deepened and shallow regions, see graph 7(a), where the higher harmonic waves are missing totally in the slow-acceleration case.

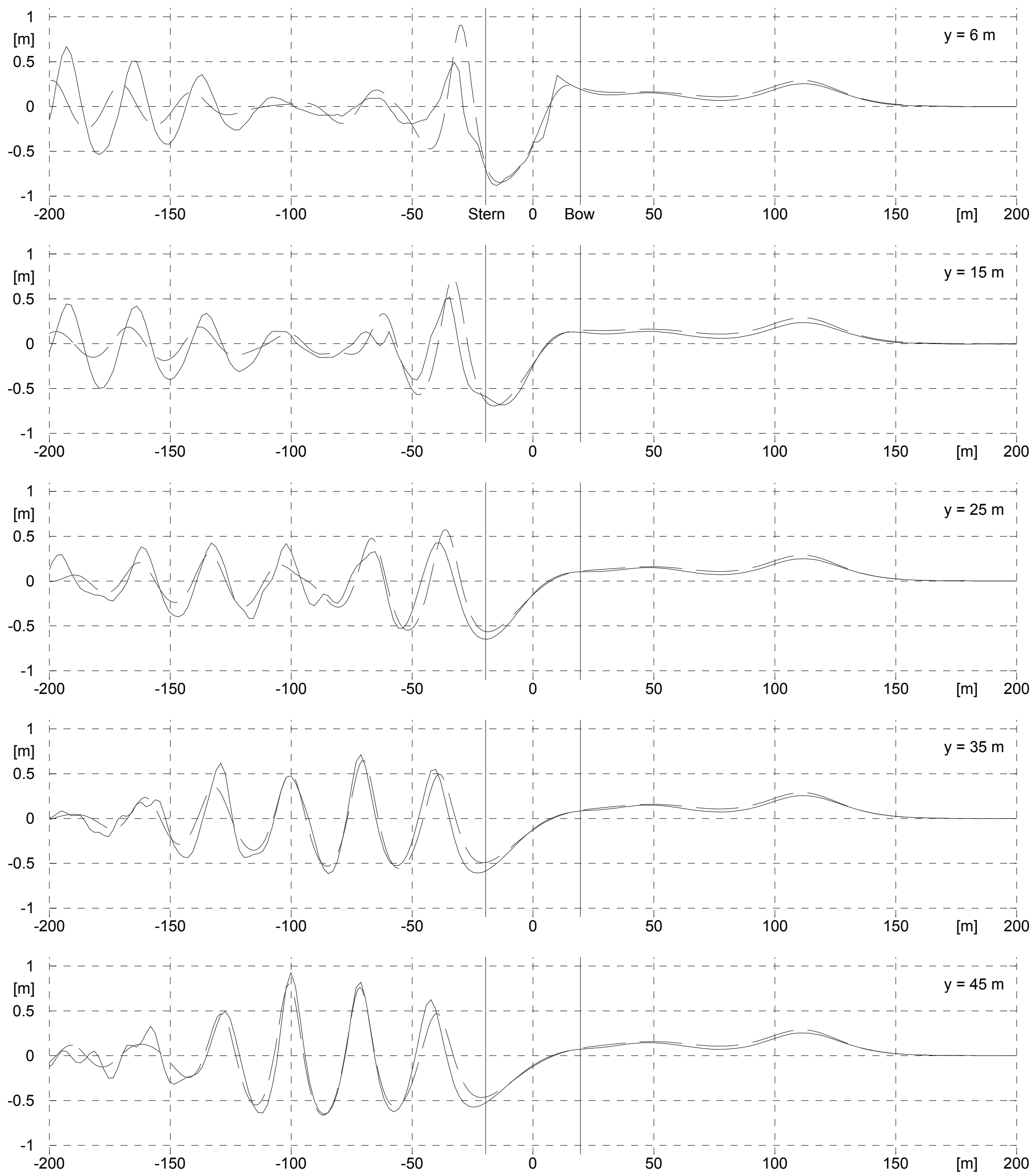
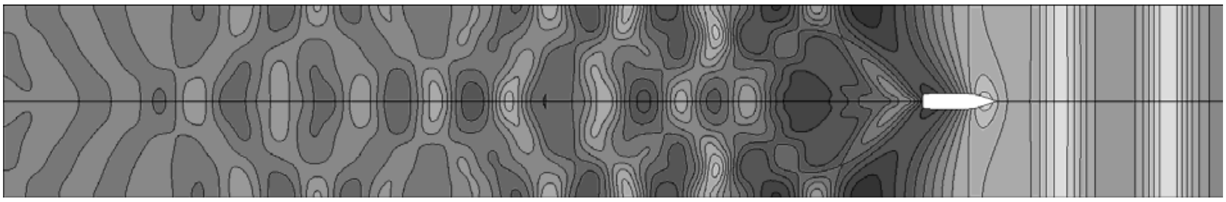
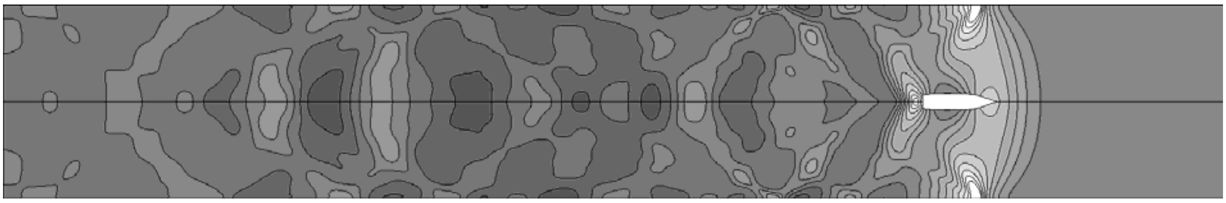


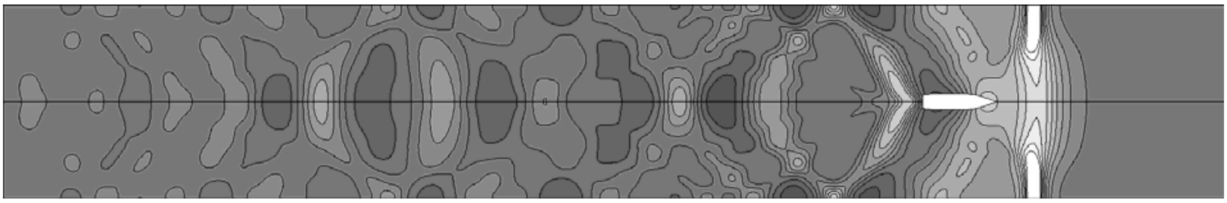

**Figure 2:** Comparison of wave records (replotted as profiles in shipbound coordinates) of the subject ferry at $F_{nh} = 0.873$ as measured in the VBD (——) and calculated by the computer program BEShiWa (– – –)



a) Contour plot of the wave pattern in a rectangular channel with uniform water-depth $h$ = 5 m



b) Contour plot of the wave pattern in a trapezoidal channel with maximum water-depth $h$ = 5 m



c) contour plot of the wave pattern in a channel with a deepened fairway ($h$ = 5 m) and shallow banks ($h$ = 3 m)

**Figure 3:** Influence of transversally varying bottom topography on the wave pattern generated by the subject inland passenger-ferry moving at constant speed $V_S$ = 7 m/s (corresponding to local $F_{nh}$ = 1 on the channel centerline)

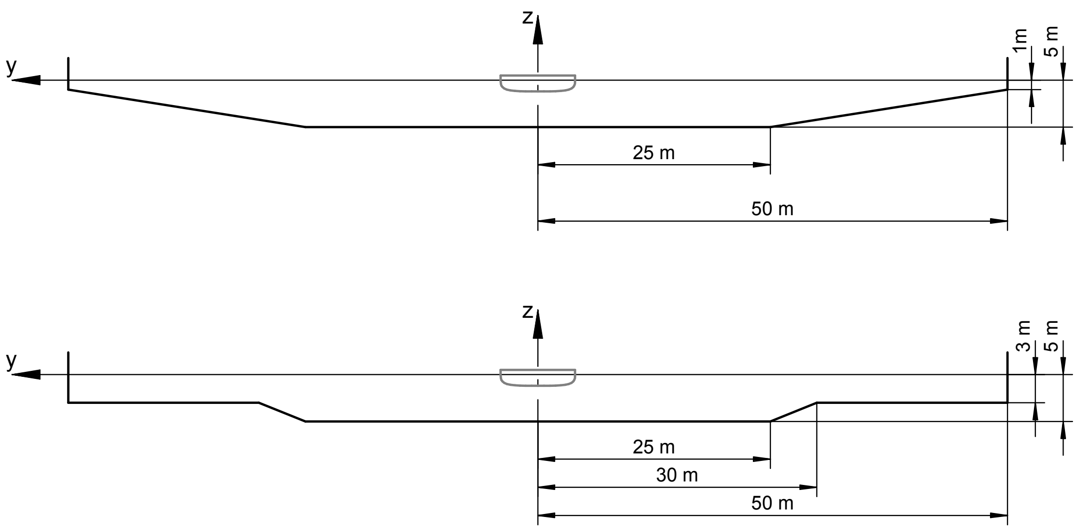



**Figure 4:** Channel section shapes for cases (b) and (c)

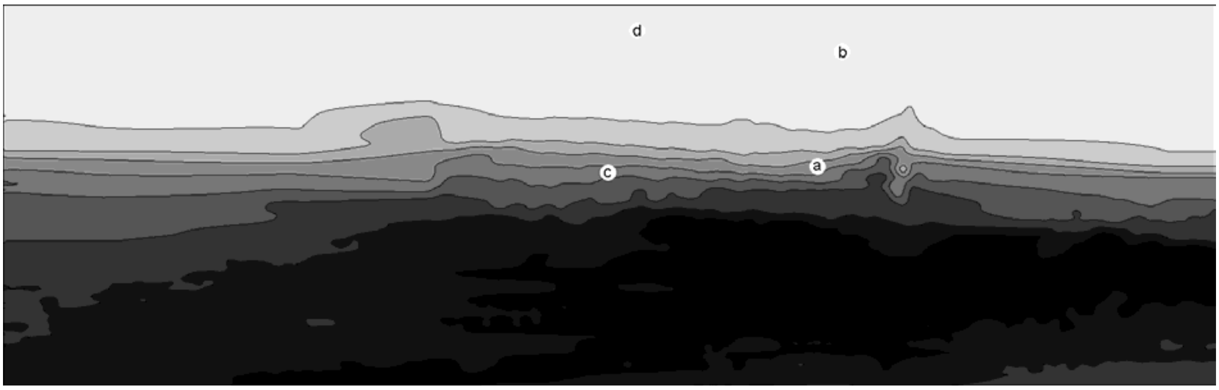
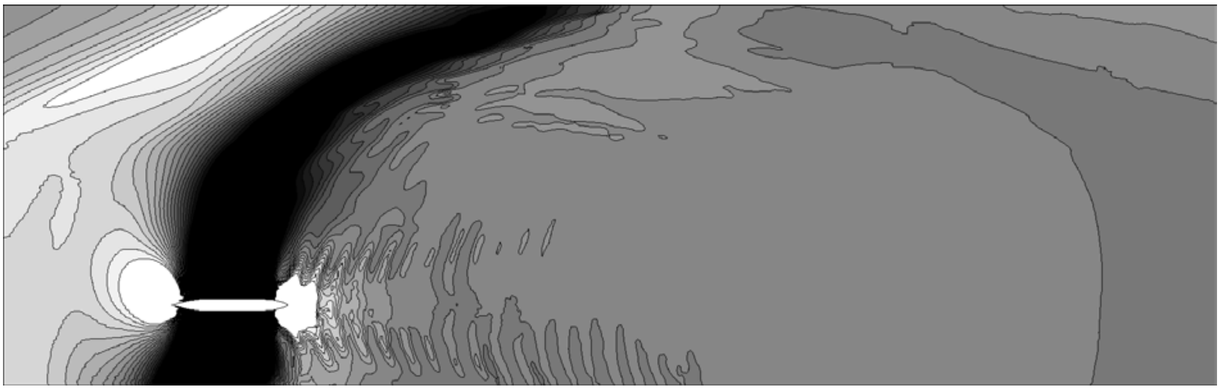
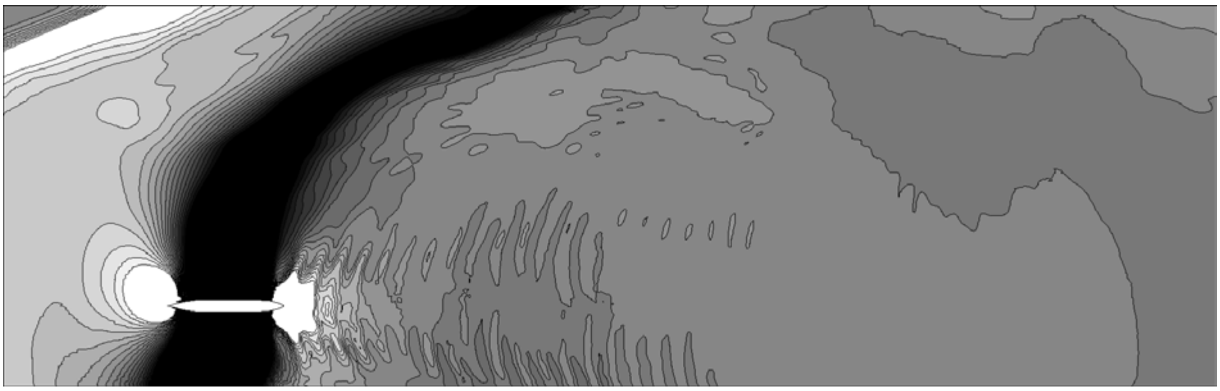



**Figure 5:** Contour plot of the bottom topography (locations of the four wave probes (a)–(d) marked)



a) Slow acceleration



b) High acceleration

**Figure 6:** Contour plots of wave patterns generated by the subject containership at a speed V=8.35 m/s in a near-shore fairway (Note: Here the direction of motion is from right to left)

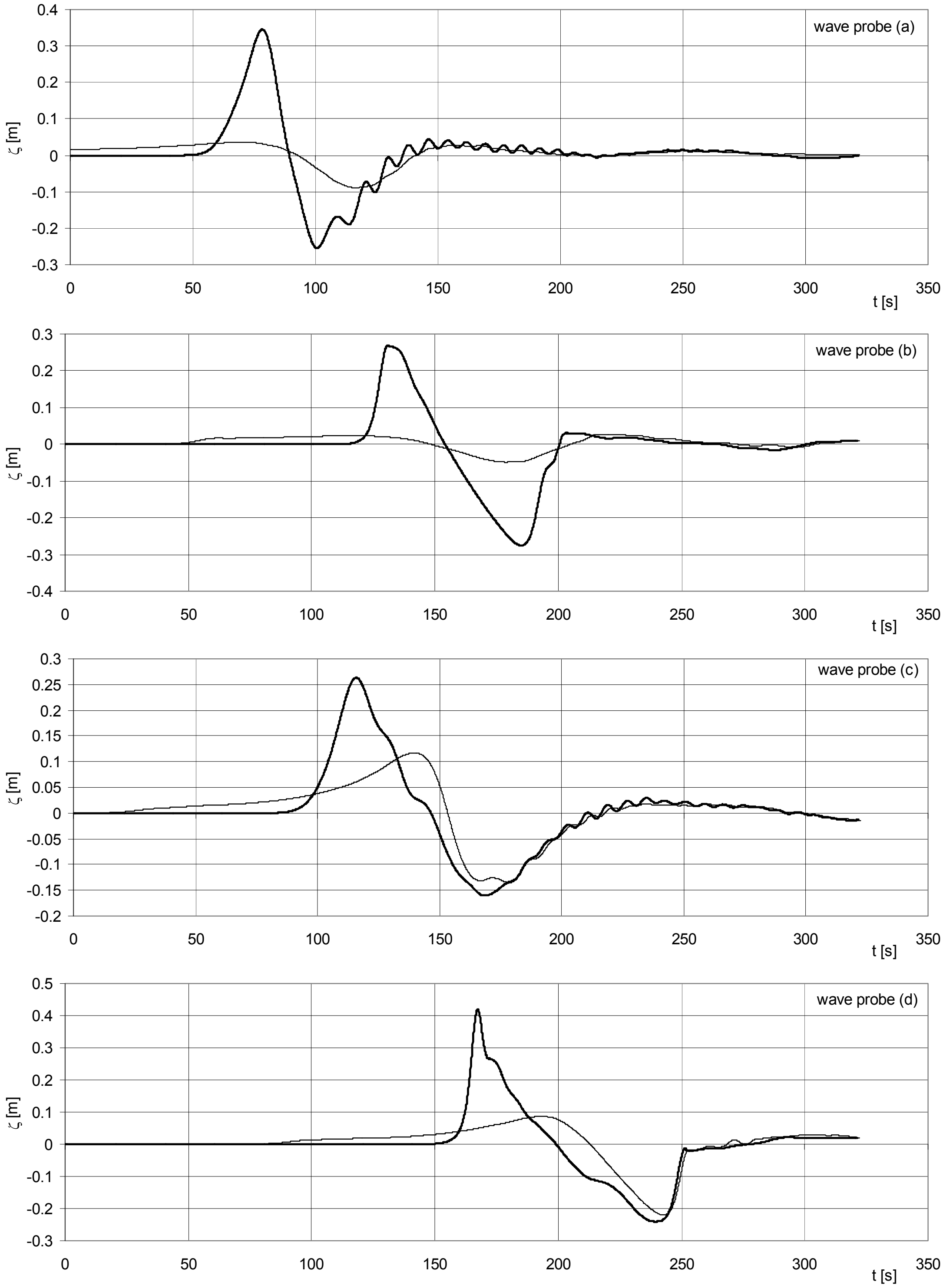



**Figure 7:** Wave records showing the influence of initial ship acceleration (—— fast, ----slow acceleration)

**Interaction of Wave Generation and Propagation with Bottom Topography**

As discussed by Feldtmann and Garner (1999), a suitable artificial ramp in a wash-sensitive region could reduce the wash. The main idea is to minimize the passing time from a supercritical speed in a shallow region to a subcritical speed in a deeper region or vice versa so that the large wave generation in the transcritical speed range may be avoided or at least reduced. Figure 9 demonstrates the wave generation and propagation over a fairway with a ramp such as shown in Figure 8. The ferry speed was assumed to be constant at 8 m/s. Before the subject ferry arrives at the ramp it moves at a supercritical speed. So the wave pattern has pronounced divergent waves accompanied by transverse waves caused by the initial acceleration, see graph (a). As soon as the ferry crosses over the ramp a strong interaction of the supercritical wave-pattern occurs with the ramp, and the wave pattern changes its form as seen in graph (b). This interaction continues until the ferry has moved far beyond the ramp, see graphs (c) to (d). Finally, there is a wave pattern typical of the subcritical speed in the deeper region, and the wake waves over the ramp almost disappear from the calculation domain due to the no-reflection condition implemented on the open truncation-boundaries to each side, see graph (e).

**Validation of Wave Propagation over Uneven Bottom**

To validate the computation of the wave propagation over an uneven bottom, a measurement performed at Delft Hydraulics (Dingemans, 1994) with a 2D bar-type bottom topography as shown in Figure 12 was numerically simulated. A wavemaker generates a harmonic wave train propagating from left to right in a wave channel. Figure 10 compares the calculated wave records with those measured by wave probes at 6 different locations. As the harmonic primary waves of period 2.02 s and amplitude 0.02 m propagate over the upward slope of the bar, the nonlinear effect increases and, hence, higher harmonics are generated, see records of wave probes located at $x = 26.04$ m and 28.04 m. These higher harmonics become quickly free over the downward slope, see records of wave probes located at $x = 30.44$ m and 37.04 m. There is remarkable agreement between calculation and measurement as long as the higher harmonics do not get free. Thereafter, a strong interaction between the primary waves and the free waves makes the latter equally important. Since the free wave has approximately half the wave length of the primary one, the dispersion relation of the classical set of Boussinesq's equations needs to be corrected, as discussed by Dingemans (1997). Similar results were obtained for primary harmonic waves of period 2.525 s and amplitude 0.029 m, see Figure 11.

## CONCLUSION

For predicting the wash waves generated by ships a method based on Boussinesq-type equations for the far-field flow and on slender-body theory for the near-ship flow yields satisfactory results. It covers all relevant effects associated with the nonlinear and unsteady nature as well as with the large-domain feature of the wash problems. Any neglect of these effects would lead to a poorer approximation.

Since the propagation of wash waves significantly depends on bottom topography, ship speed and motion history, any measures for reducing wash waves deduced from computational predictions need to be validated by experiments. At the same time, however, this strong dependence opens up possibilities of formulating suitable criteria for safe ship operation (speed and distance to shoreline or river bank) as well as for effective fairway management (construction and maintenance).

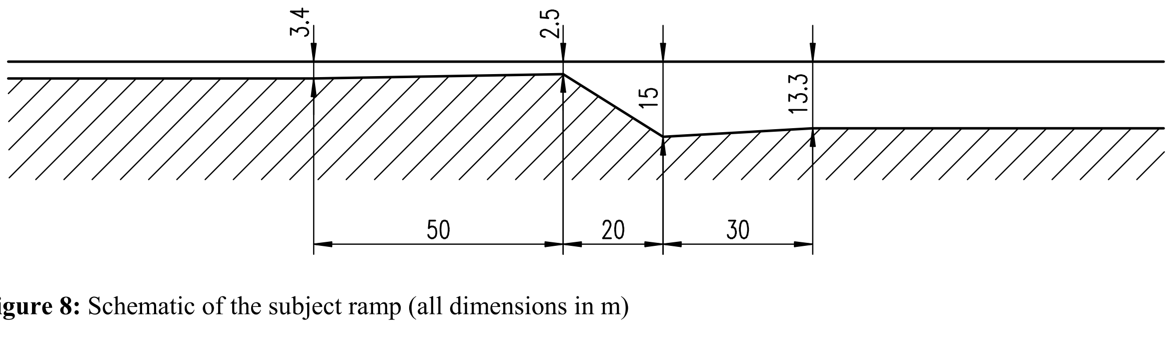
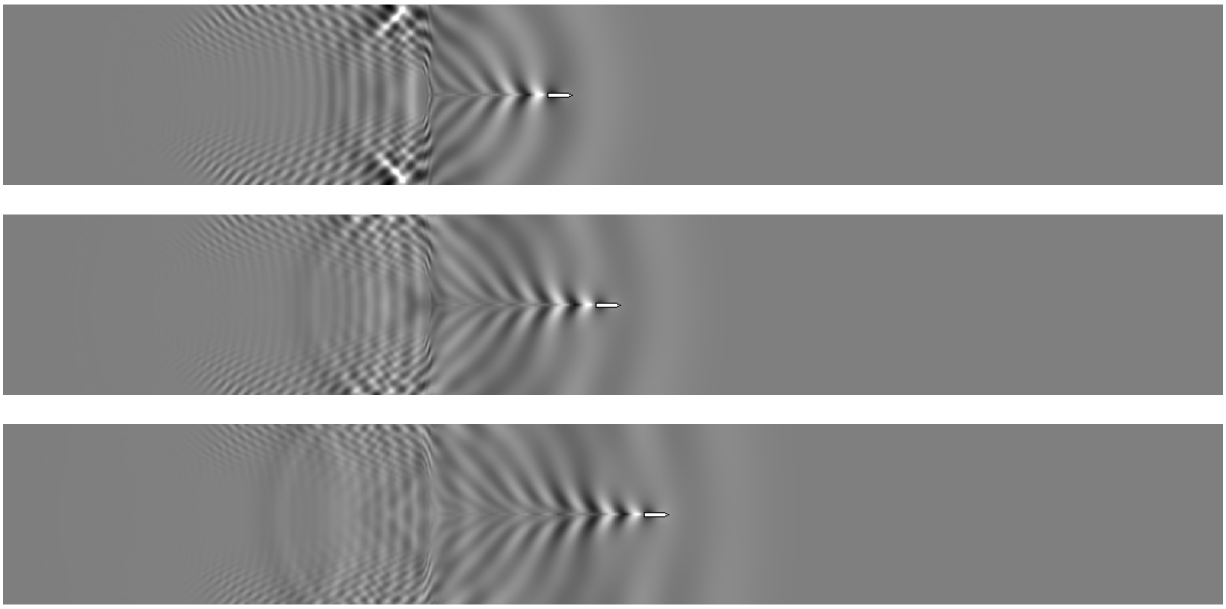



**Figure 8:** Schematic of the subject ramp (all dimensions in m)

(a) wave pattern generated by the subject ferry in the shallow region before the ferry moves over the ramp

(b) interaction of the supercritical wave-pattern with the ramp as the ferry crosses over the ramp

(c) wave pattern as the ferry leaves the ramp behind



(d) evolution of the wave pattern while the ferry moves beyond the ramp

(e) wave pattern of the ferry at subcritical speed in the deeper region

**Figure 9:** Evolution of the wave pattern generated by the subject inland passenger-ferry moving at a constant speed of 8 m/s over a fairway with a ramp as shown in Figure 8.

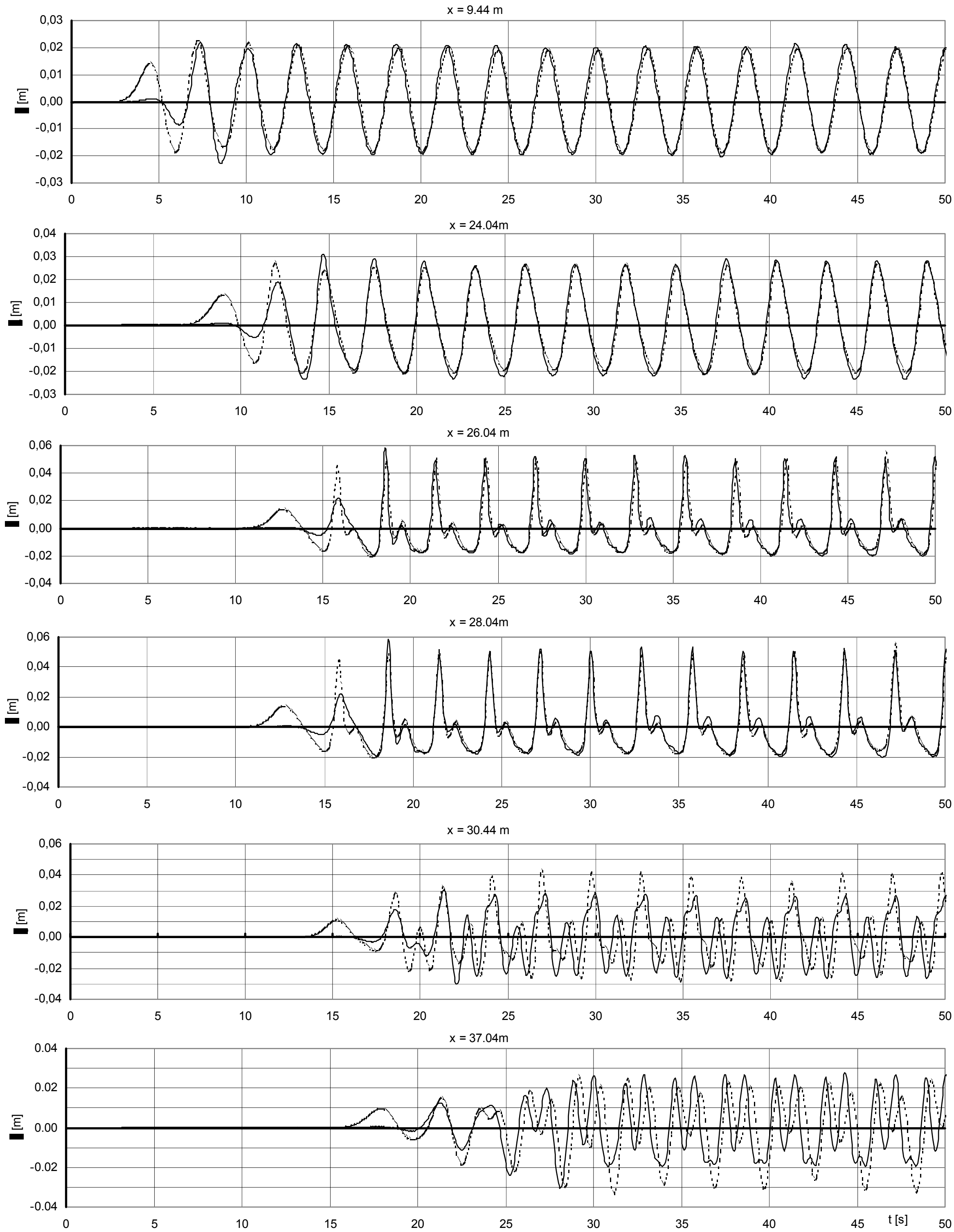



**Figure 10:** Records of wave elevation at 6 different wave-probe locations for an initially harmonic wave of period 2.02 s and amplitude 0.02m propagating over an uneven bottom (see Figure 12) as measured by Dingemans (——) and calculated using BEShiWa (- - -)

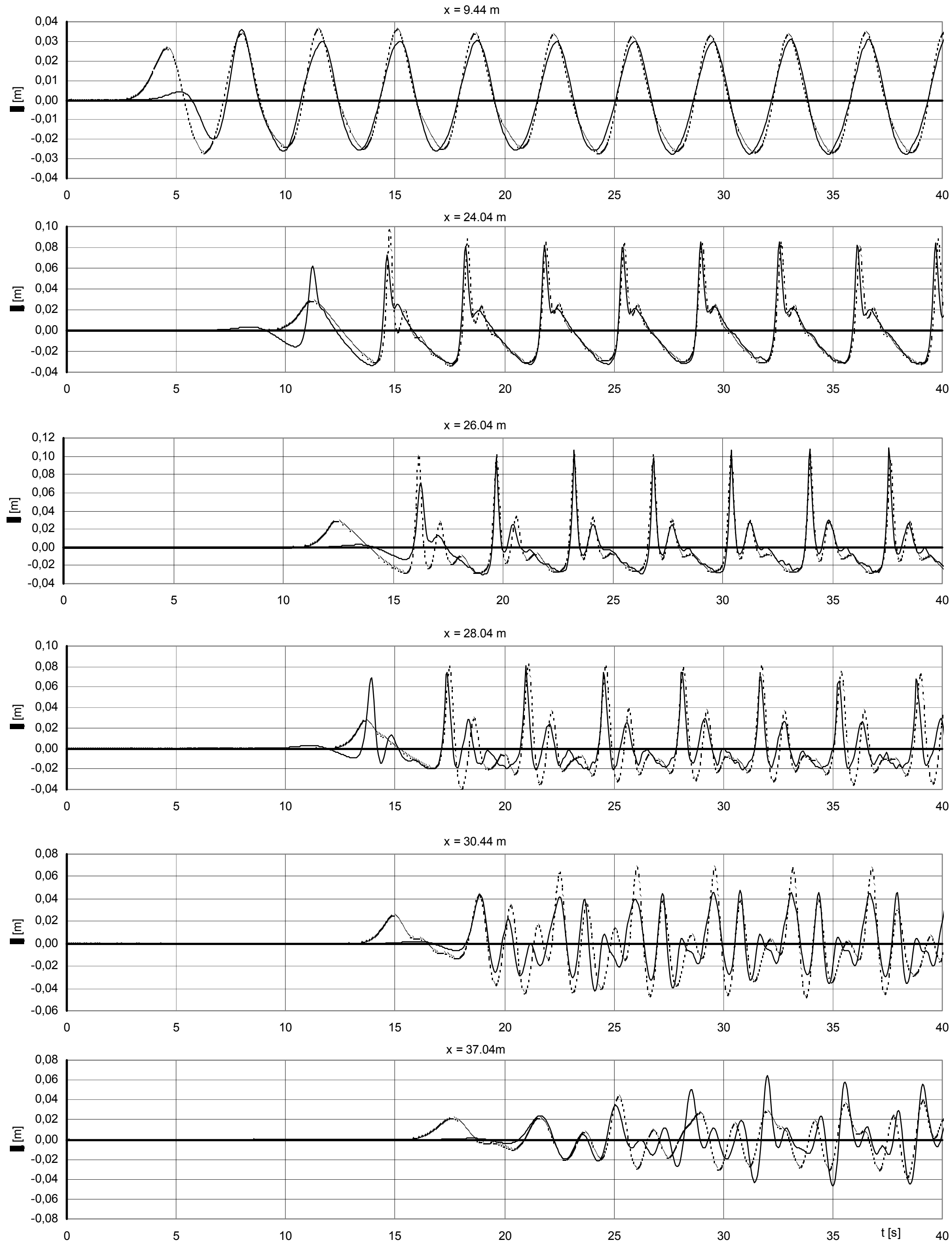



**Figure 11:** Records of wave elevation at 6 different wave-probe locations for an initially harmonic wave of period 2.525 s and amplitude 0.029 m propagating over an uneven bottom (see Figure 12) as measured by Dingemans (—–) and calculated using BEShiWa (- - -)

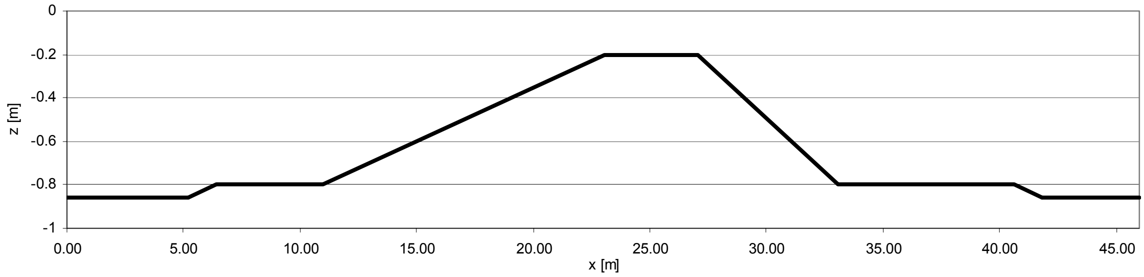



**Figure 12:** The two-dimensional bar-type bottom topography investigated by Dingemans

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